Math 6710 Applied Linear Operators and Spectral Methods
Fall 2020 Syllabus

Instructor: Fernando Guevara Vasquez (he, him, his)

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Zoom handle: [https://utah.zoom.us/j/94275898082](https://utah.zoom.us/j/94275898082) (please see Canvas for the passcode)

Office: JWB 330

Phone: 801-581-6131

Accessibility and Support: The preferred methods for reaching me are (in order of preference/reactivity): a message on Canvas or via email. I answer email daily, usually in the mornings.

Office hours: MW 2-3 or by appointment using the Zoom handle above. Please check Canvas for any changes or cancellations.

Course Description:
This is an introduction to functional analysis with an eye on applications and is intended for graduate students that have had real analysis (e.g. the Math 3210-3220 series or equivalent OR preferably Math 5210 or equivalent).

For mathematics graduate students:
To earn a “high-pass” in this class you need a A. To earn a “pass” you need a A− or B+. Earning a “pass” or “high-pass” in this class can dispense you from taking the preliminary / written qualifying exam associated with this class. For more details, please see the [graduate bulletin](https://example.com), which is the authoritative document on this matter. Also do not hesitate to ask the instructor about this.

Course Details:

Course Type: Interactive Video Conferencing (IVC)

Location & Meeting Times: Canvas. TuTh / 2pm-3:20pm. The Zoom link is above and the passcode will be given in Canvas.

Attendance & Punctuality: Attendance to the video lectures is expected but not mandatory. For those that cannot make it class notes and videos of the lectures will be regularly posted on Canvas. However to get the best learning experience, you are encouraged to attend and actively participate in the lectures. Examples of participation are to ask questions, ask to re-explain or review some concept etc... Lectures should not be a monologue, participating is relatively easy (via e.g. chat or audio) and makes the lecture more valuable for everyone involved.

COVID-19 Considerations: Students must self-report if they test positive for COVID-19 via [coronavirus.utah.edu](https://coronavirus.utah.edu)

Course Materials: All materials for this course (including class notes and problem assignments) are copyrighted. Do not distribute or share course resources without instructor permission.


Other references:

Technical Requirements

- Students are expected to be computer literate and [Canvas](https://canvas.utah.edu) and zoom navigation skills are expected. Knowledge and navigation of canvas and zoom is critical to access all features and resources of this course.
• This class includes synchronous components (live Zoom sessions). A stable connection and adequate bandwidth is needed. Lectures will be recorded for your convenience. Please do reach out to the instructor if you anticipate having problems attending the lectures in this fashion.

Syllabus subject to change: This syllabus is meant to serve as an outline and guide for our course. Please note that I may modify it with reasonable notice to you. I may also modify the Course Schedule to accommodate the needs of our class. Any changes will be announced in class and posted on Canvas.

Content Overview: This class is an introduction to functional analysis. Most of the class focusses on how do results of linear algebra change when we are in infinite dimensions. To give you a taste of how interesting things get: in normed finite dimensional vector spaces we know that all closed and bounded sets are compact, however this no longer holds in infinite dimensions! We will work our way through culminating with spectral theory, i.e. studying particular linear operators (matrices in finite dimensions) that are diagonalizable. We then use the tools of functional analysis to understand distributions or “generalized functions”.

Course expected learning outcomes: After this class, students should be familiar with the concepts outlined in the detailed syllabus. Students are expected to be able to apply these concepts to different situations, understand the proofs of such concepts and also be able to reproduce their derivation, with possibly some guidance. This is reinforced by proving results during regular homework problems and by applying the results to a variety of examples.

Course Design: The instructor will deliver online lectures via Zoom during the usual class times. Homeworks are assigned weekly. Although it may be easy to find solutions online, you will not learn anything from it. It is better figure the problems out on your own or with a group of other students. It may be unpleasant and frustrating to hit dead-ends, make mistakes and spend a lot of time on this, but it is the only effective way I know to learn mathematics and that is exactly how research feels like (most of the time). To help you, there will be two regularly scheduled virtual office hours per week. The instructor is also available for virtual meetings with prior notice. There will be two exams: a midterm and a final, and both will evaluate how well you know the concepts learned in class, how well you can apply them and whether you can prove certain results (perhaps with some guidance).

Evaluation: All assignments (homeworks, midterm and final) are to be submitted electronically using Gradescope. Please check the Canvas webpage for more info about this. In particular, this means you need a means of scanning your assignment (such as a scanner or smartphone) OR you may also handwrite your assignments on a tablet. Typing up your assignments is also acceptable but NOT necessary. Since the midterm and final are both time-limited, you should not attempt to typeset these assignments.

• Homeworks – 50%: Problems are assigned weekly and a (clearly indicated) subset of them will be graded. The two lowest homework grades will be dropped to compute the homework average.

• Midterm – 20%: 80min, closed book/notest, Tue Oct 6 2pm-3:20pm (tentative). The exam will be proctored remotely using Zoom.

• Final – 30%: 120min, closed book/notest, Wed December 9 1-3pm (per university schedule). The exam will be proctored remotely using Zoom.

Remote proctoring: The exams in this class will proctored remotely over a Zoom video call. This means that you will need a quiet room for the duration of the exam, a stable internet connection and a device with webcam that is able to run Zoom so that the instructor can see you and your work area while taking the exam. Please contact the instructor if you anticipate any problems with this or if you need any special accommodations.

Academic Code of Conduct: Students are encouraged to review the Student Code for the University of Utah: [https://regulations.utah.edu/academics/6-400.php](https://regulations.utah.edu/academics/6-400.php) In order to ensure that the highest standards of academic conduct are promoted and supported at the University, students must adhere to generally accepted standards of academic honesty, including but not limited to refraining from cheating, plagiarizing, research misconduct, misrepresenting one’s work, and/or inappropriately collaborating. A student who engages in academic misconduct as defined in Part I.B. may be subject to academic sanctions including but not limited to a grade reduction, failing grade, probation, suspension or dismissal from the program or the University, or revocation of the student’s degree or certificate. Sanctions may also include community service, a written reprimand, and/or a written statement of misconduct that can be put into an appropriate record maintained for purposes of the profession or discipline for which the student is preparing.

Grade scale: If X is your percentage grade, then \( \{ X \geq 93\% \Rightarrow A, X \geq 90\% \Rightarrow A-, X \geq 87\% \Rightarrow B+, X \geq 83\% \Rightarrow B, X \geq 80\% \Rightarrow B-, X \geq 77\% \Rightarrow C+, X \geq 73\% \Rightarrow C, X \geq 70\% \Rightarrow C-, X \geq 67\% \Rightarrow D+, X \geq 63\% \Rightarrow D, X \geq 60\% \Rightarrow D-, X < 60\% \Rightarrow E \} \). Letter grade assignments can be changed at the discretion of the instructor.

Communication
All course materials, such as lecture videos, lecture notes, assignments, solutions, grades, etc. will be posted on the Course Canvas site. Class announcements will be done via email through the Canvas server. You will be responsible for any information contained in them as well as the information announced in class.

It is your responsibility to also regularly check your Umail (make sure you set up forwarding if you do not check it regularly), your Umail is the only way for me to communicate privately with you, there will be occasions during the semester that we may need to reach out to you individually (e.g. regarding a grade or assignment) and it is in your best interest to respond promptly.

Feel free to contact me by email for questions at fguerva@math.utah.edu. I will do my best to answer emails promptly. I would like to encourage you to email me only if it is something personal that requires individual attention, if instead you have questions about logistics of the class, course material and assignments, and anything else your classmates may wonder as well, please post a question on the Discussions Board instead. This way the information is shared quickly to the entire class, and each of you can benefit from seeing other classmates’ questions.

Additional Policies and Resources

The Americans with Disabilities Act: The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability & Access, 162 Olpin Union Building, 801-581-5020. CDA will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in alternative format with prior notification to the Center for Disability & Access.

Addressing Sexual Misconduct: Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a Civil Rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran’s status or genetic information. If you or someone you know has been harassed or assaulted on the basis of your sex, including sexual orientation or gender identity/expression, you are encouraged to report it to the University’s Title IX Coordinator; Director, Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, or to the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to police, contact the Department of Public Safety, 801-585-2677 (COPS).

Campus Safety: The University of Utah values the safety of all campus community members. To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit safe.utah.edu

University Counseling Center: The University Counseling Center (UCC) provides developmental, preventive, and therapeutic services and programs that promote the intellectual, emotional, cultural, and social development of University of Utah students. They advocate a philosophy of acceptance, compassion, and support for those they serve, as well as for each other. They aspire to respect cultural, individual and role differences as they continually work toward creating a safe and affirming climate for individuals of all ages, cultures, ethnicities, genders, gender identities, languages, mental and physical abilities, national origins, races, religions, sexual orientations, sizes and socioeconomic statuses.

Office of the Dean of Students: The Office of the Dean of Students is dedicated to being a resource to students through support, advocacy, involvement, and accountability. It serves as a support for students facing challenges to their success as students, and assists with the interpretation of University policy and regulations. Please consider reaching out to the Office of Dean of Students for any questions, issues and concerns. 200 South Central Campus Dr., Suite 270. Monday-Friday 8 am-5 pm.
Detailed syllabus:

1. **Introduction to function spaces (Chapters 1 and 2)**
   - Metric spaces: convergence, completeness
   - Continuity of functions
   - Separability
   - Contraction mapping principle
   - Vector spaces
   - Banach spaces
   - Compactness
   - $L^p$ spaces (without measure theory)

2. **Linear operators (Chapters 2 and 4)**
   - Linear operators
   - Bounded linear operators
   - Linear functionals
   - Dual spaces
   - Compact operators
   - Mention Hahn-Banach Theorem, Open Mapping theorem, Closed Graph Theorem and consequences
   - Weak and weak$^*$ convergence
   - Reflexive spaces
   - Fredholm alternative

3. **Hilbert spaces (Chapter 3)**
   - Inner product spaces
   - Orthogonal projections
   - Orthonormal sets
   - Linear functionals and bilinear forms
   - Riesz representation theorem
   - Lax-Milgram theorem (supplementary—not in text)
   - Adjoint operator
   - Fredholm alternative in Hilbert spaces

4. **Spectral theory (parts of Chapters 7, 8, 9)**
   - Resolvent and spectrum
   - Basic results for bounded linear operators
   - Spectral properties of compact operators
   - Bounded self-adjoint operators
   - Spectral theorem for compact self-adjoint operators
   - More general spectral representations

5. **Distributions (supplementary text)**
   - Spaces of test functions
   - Definition of distributions
   - Operations on distributions
   - Fourier transform and tempered distributions