Instructor: Sean Howe

Contact: Questions and concerns should be communicated in office hours or via email to sean.howe@utah.edu. In most circumstances, I will respond to emails before 11:59pm the next business day (business days = all days except Saturdays, Sundays, and university holidays).

Meetings and office hours: Classes and office hours will be held on Zoom; meeting information will be posted on Canvas at the start of the semester. Individual meetings are available by request.

Text: There is no textbook for the class. Online resources will be posted to Canvas.

Attendance and punctuality: Synchronous attendance is optional, however, registered students missing more than one synchronous class session in a two-week period must contact the instructor within one week of the second absence to discuss their plan for keeping up with the course content asynchronously.

Course materials: Recorded lectures and notes will be made available through Canvas. All materials for this course are copyrighted; do not distribute course resources without instructor permission.

Course design: The level of this course lies somewhere between a first-year graduate course and a topics course. The synchronous sessions will contain a mix of lecture, problem solving, and discussion.

Course advertisement: An elementary introduction to p-adic Hodge theory from a modern perspective. Classical Hodge theory studies the relation between singular and de Rham cohomology for complex algebraic varieties. P-adic Hodge theory studies the relation between the p-adic etale, de Rham, and crystalline cohomology of p-adic varieties. Introduced by Tate in the 60s then developed by Fontaine, Faltings, and others in the 70s and 80s, p-adic Hodge theory became a pillar of modern number theory in the 90s through its central role in the study of modularity of Galois representation (Wiles, Fontaine-Mazur, etc.).

Over the past decade, the language and scope of p-adic Hodge theory has undergone a fundamental transformation through the introduction of the Fargues-Fontaine curve and Scholze’s theory of perfectoid spaces. This course will present an elementary introduction to p-adic Hodge theory that takes these advances into account. We will only assume knowledge from graduate algebra (at the level of 6310-6320).

In particular, we will not assume prior knowledge of algebraic number theory, etale cohomology, or the p-adic numbers. Instead, basic concepts in these areas will be developed as needed, with an emphasis placed on illustrations through concrete examples derived from elliptic curves. Some background in the theory of algebraic curves and/or Riemann surfaces will be helpful, but not necessary, for the course.

Learning goals:
After the course, students will be able to
1. Explain why the p-adic representation theory of the Galois group of $\mathbb{Q}_p$ is richer than the l-adic representation theory when $l$ is not p.
2. Use Witt vectors to construct the p-adic numbers and related rings as “Laurent series in the variable p.”
3. Decompose vector bundles on curves using the slope formalism.
4. Identify the classical comparison theorems in p-adic Hodge theory in terms of trivializations of equivariant vector bundles on geometric loci of the Fargues-Fontaine curve.
5. Identify the difference between crystalline, semi-stable, and de Rham representations, as illustrated by geometric examples using elliptic curves.

The course will also include survey material on more advanced topics.

Prerequisites: Ring and Galois theory at the level of Math 6310-6320 (cf. course contents in the graduate bulletin (http://www.math.utah.edu/dept/gradbull.pdf)).
**Exams/Homework/Grading:** There are no exams for this course. Registered students will be required to present an example of their choice during office hours once in September, once in October, and once in November, or submit written examples if they are unable to attend office hours. Final grades will be determined based on completion of this task and adherence to the attendance policy described above.

**Academic Integrity:** All university policies regarding ethics and honorable behavior apply to this course. Students are encouraged to review the Student Code for the University of Utah:
https://regulations.utah.edu/academics/6-400.php

**COVID-19 Considerations:** Students must self-report if they test positive for COVID-19 via coronavirus.utah.edu.

**The Americans with Disabilities Act:** The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability & Access, 162 Olpin Union Building, 801-581-5020. CDA will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in alternative format with prior notification to the Center for Disability & Access.

**Addressing Sexual Misconduct:** Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a Civil Rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information. If you or someone you know has been harassed or assaulted on the basis of your sex, including sexual orientation or gender identity/expression, you are encouraged to report it to the University’s Title IX Coordinator; Director, Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, or to the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to police, contact the Department of Public Safety, 801-585-2677(COPS).

**Campus Safety:** The University of Utah values the safety of all campus community members. To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit safeu.utah.edu