Syllabus

History and Philosophy of Mathematical Logic

PHIL 3013: class # 17574
Designation: HF
Credit: 3 Units

Instructor: Dr. Chapman Waters
Office: CTIH 441
Office Hours: TBD
Email: chapman.waters@utah.edu
Class Location: GC 2575
Class Time: MWF 9:40am - 10:30am
Prerequisites: none, but a previous course in logic would be ideal.

Course Description

This course will examine the history and philosophy out of which contemporary mathematical logic developed. Historical figures that will receive special emphasis include Cantor, Frege, Russell, Hilbert, and Gödel. Philosophical emphasis will be on issues pertaining to the metaphysical and epistemological foundations of mathematics and logic. For a more in depth characterization of topics covered, see “Learning Outcomes and Course Structure” below.

Learning Outcomes and Course Structure

Students will gain an understanding of various issues that logicians are currently interested in by looking at the historical developments that gave rise to them. Equally important is that students will gain familiarity with the philosophical motivations and/or controversies that are intwined with these developments.

Specifically, the course will be organized around the following figures and students will gain an understanding of their logical/mathematical/philosophical contributions:

GEORG CANTOR, INFINITY, AND SET THEORY
Set theory would have been a footnote in the history of mathematical logic, if not for Cantor’s amazing discovery of the hierarchies of larger infinities. We will examine this discovery, its history, and surrounding philosophical issues.

FREGE, RUSSELL, AND LOGICISM
This portion of the course will start with Gottlob Frege’s “Begriffsschrift” (1879), that is to say, the invention of modern quantificational logic. Particular points of focus will be on Frege’s understanding of the purpose of his notation (ie, as a scientific tool), Frege’s conception of the discipline of logic (ie, as the science that studies “the laws of truth”), and Frege’s attempt to establish Logicism (ie, the thesis that mathematics is a branch of logic). We will trace these ideas onto Bertrand Russell – who discovered the paradox that brought down Frege’s project – by considering Russell’s subsequent vindication of Logicism and the Ramified Theory of Types.
HILBERT’S PROGRAM
With the paradoxes of set theory, and the subsequent troubles for Logicism, another important development came with David Hilbert. “Hilbert’s Program” provided the “formal” approach to axiomatic systems needed to overcome the paradoxes, account for the true nature of the infinite, and it provides the historical antecedent of the now widely accepted “model-theoretic” conception of logic.

GÖDEL’S INCOMPLETENESS THEOREMS
We finish the course by considering Gödel’s Incompleteness Theorems. Gödel’s theorems establish that no final systematization of many important areas of mathematics is attainable. This groundbreaking achievement introduced new methods and problems for logico-mathematical investigation — and it set the agenda for projects which are still underway today. After gaining a rudimentary understanding of how Gödel proved his theorems, we will take up some of the issues that it raises: Does Gödel’s result have devastating effects on Formalism, Logicism, and other foundational programs in mathematics? Does it establish the truth of mathematical Platonism, the impossibility of genuinely intelligent machines (strong AI), or even the existence of God?!

Required Texts

There are two books for the course:


All remaining readings will be available on the course CANVAS page.

Grading

Grades will be given on a 0-100 point grading scale:

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<th>Score Range</th>
<th>Grade</th>
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<tr>
<td>100-93:</td>
<td>A</td>
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<td>92-90:</td>
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Final grades will be determined by reading questions, Canvas surveys, online discussion, and four short writing assignments. These will be weighted as follows:
15% Reading Questions
10% Canvas Surveys
25% Online Discussion
50% Four Short Writing Assignments

READING QUESTIONS
Grappling with historical texts is an important part of this course. So, for FOUR of the assigned readings, you will submit a question (or two) to exhibit that you have done the reading and that you have thought about the material enough to have some kind of response to it. (It could be an ‘I don’t understand’ question; but it must be an informed ‘I don’t understand’ question. In other words, the question would need to diagnose why you are confused, not just that you are confused.) Length of questions should not exceed two full paragraphs. Grades will be given on a two-point scale: if your reading question(s) demonstrate that you read the material and engaged with it non-superficially, you’ll get full points; you’ll get one point for questions that the instructor deems “acceptable” and no points for questions that the instructor deems “unacceptable.” If you fail to get full points on a reading question that you submit, you may turn in additional reading questions in an attempt to boost your grade.

It is your responsibility to ensure that you’ve completed all four by the end of the semester. You will turn your question in through the Canvas website, so that your responses will be documented online. NOTE: you must turn in questions about a particular text before we stop discussing it in class.

CANVAS SURVEYS
I will administer “surveys” through Canvas somewhat regularly throughout the semester. These will function very much like polls in that you’ll be asked for your opinion, viewpoint, input, etc., about a variety of class related items. Unlike polls, however, Canvas surveys will be graded. Grading for surveys will be all-or-nothing: you will receive full points for any survey that you complete and no points for any survey that you fail to complete. Take note: each survey will have a due date and you will not be permitted to complete a survey after its due date has passed. It is your responsibility to ensure that you complete each survey before the time at which it is due. Moreover, you are required to complete each and every survey that is administered through Canvas. [At present the number of surveys that will be given, as well as their due dates, are undetermined. Announcements with the relevant information will be given (in class and on Canvas) as the semester progresses — so it’s crucial that you’re keeping track of what’s happening on the course webpage and in class!] The nature of survey questions will be varied: some will be used to track your intuitions on a topic both before and after it has been covered in class; others will track student views about course topics, the best or worst way to respond to them, or similar things; some will ask for input about the structure of the class, what’s working and what’s not, and potential changes to the course; still others will ask about your pedagogical needs and/or preferences, among other things.

ONLINE DISCUSSION
There will be at least ten discussion threads posted on the Canvas course webpage this semester, but very possibly more. You will be required to participate on at least five different discussion threads before the end of the semester. Of course, there is no limit to how many discussion threads students may contribute to. Only those five with the highest scores will count toward your final grade. It is your responsibility to ensure that you’ve engaged with at least five different discussions by the end of the semester. General information about online discussion, along with a grading rubric, will be provided at the top of the Canvas course homepage. The nature of online discussions
will be varied. Some will give you a prompt to respond to; others may require you to discuss a youtube video or video lecture; some may require you to conduct group discussions or video discussions with your peers; still others may require you to complete “group projects” or to collaborate on Short Writing Assignments with others (see below). It’s also possible that one or two of the discussions will be mandatory. Details and additional information will be provided to students (through Canvas) as needed.

**FOUR SHORT WRITING ASSIGNMENTS (SWAs)**
There will be a total of six SWAs provided during the semester. You must turn in at least four of these. You may turn in all six, of course, but only those four with the highest scores will count toward your final grade. That is to say, you’ll be given two “mulligans”: if you don’t like your grade on one (or two) of the first four assignments, you’ll have the opportunity to make them up later. So, it’s in your best interest to complete the first three or four SWAs; if you don’t, you’ll be stuck with the grades you get on the assignments that remain. There will be different requirements for each assignment. Detailed instructions will be provided to you in class and/or on Canvas a reasonable amount of time before they are due.

**COVID-19 and Course Policies**
As you’re aware, we are living in strange and difficult times. COVID-19 will certainly have its impact on how the semester unfolds. And I’m going to be honest: I don’t know exactly how that impact will affect us here, in this class. But here’s something that I do know: in our effort to get the most out of this class, to make it the best possible learning experience in light of current circumstances, we will need to work together. One thing that will be crucial is that we’re prepared to be dynamic, ready to switch things up as we go. In the spirit of giving some order to the chaos, I’ve outlined (below) some relevant strategies, guidelines, and course policies that we will adopt in this course. Among other things, these are designed to help us accomplish our academic goals while navigating the evolving COVID crisis.

**Class Meetings** — According to the University’s stipulations, this qualifies as a “hybrid” course. Officially, this means that the number of face-to-face hours that we would otherwise spend together this semester will be reduced via the use of remote learning technologies. So, this semester, this course will have some mix of in-person and online learning modalities. However, what that ultimately ends up looking like could go any number of ways. It’s my opinion that we should make things work for us, limiting the number of face-to-face hours in ways that we — as a group — think would work best for us. To that end, we’ll adopt an ongoing open dialogue about matters related to course structure this semester — about what’s working for us, and what’s not, and we’ll be willing to change things “on the fly” to accommodate this particular class’s needs and preferences. Importantly, though, also take note of the following: per University policy, no student will be required to attend any face-to-face class meetings: students may elect not to attend class in-person with no penalty and do not need to seek an official accommodation to participate online, rather than in-person. As such, students will be given access to most course materials (e.g., lecture notes) online and, whenever possible, face-to-face meetings will be live-streamed (and/or recorded) so that students may attend in-person classes remotely.

**A Few Options for Hybrid Course Structure:**
(1) Hold synchronous online discussion seminars — via Zoom — once every week or two in place of regular class meetings;

(2) use “flipped classroom model,” conducting some number of lectures online (e.g., via asynchronous video lectures) and reserving f2f class meetings exclusively for discussion, Q & A, or other group learning activities;

(3) rotational f2f attendance...among others

Seating Chart — All face-to-face students will be required to sit in the same place for each class. Having a seating chart will simplify contact tracing should any student become infected.

Transitioning from F2F to Online — There will be times when we are required to transition from a face-to-face class-format to a fully online format. According to the current schedule, this semester we will go fully online from Sep. 27 through Oct. 11 (fall break has been cancelled). Moreover, all classes will move to a fully online format after Thanksgiving break until the end of the semester (Nov. 30 - Dec. 3). In addition, we will be required to go fully online if/when there are any spikes in COVID cases. We must be ready for this, as such transitions are likely to occur with only hours of notice. In this course, a fully online course format will likely include a combination of synchronous and asynchronous elements. More details to come.

Face Masks — Face coverings are required in all in-person classes for both students AND faculty. Based on CDC guidelines, the University requires everyone to wear face coverings in shared public spaces on campus, including our classroom. As a reminder, when I wear a face covering, I am protecting you. When you wear a face covering, you are protecting me and all of your classmates. If you forget your face covering, I will ask you to leave class to retrieve it. If you repeatedly fail to wear a face covering in class, I will refer you to the Dean of Students for a possible violation of the Student Code. Note that some students may qualify for accommodations through the Americans with Disabilities Act (ADA). If you think you meet these criteria and desire an exception to the face covering policy, contact the Center for Disability and Access (CDA). Accommodations should be obtained prior to the first day of class so that I am notified by CDA of any students who are not required to wear a face covering.

Canvas — The Canvas course webpage is your contact point for this course, and so, it’s important that you familiarize yourself with it (for a tutorial page, visit the link pasted below). The “homepage” will provide a detailed course schedule that will be updated as we go. I will also use the Canvas homepage to distribute all course materials (course schedule, readings, handouts, study guides, etc.). Moreover, all coursework will be turned in through Canvas.

Canvas Tutorial: https://community.canvaslms.com/docs/DOC-10701

Self-Reporting — Students must self-report if they test positive for COVID-19 via utah.edu.
Attendance Policy — None. That said, if you do not attend f2f class meetings in-person, I strongly recommend that you attend remotely. See the “Class Meetings” paragraph above.

Coursework Deadlines: In the absence of justifiable prior arrangements or cases of serious, unexpected and documented medical emergencies or bereavement, coursework must be turned in as scheduled. For circumstances that are not emergencies, students must send the instructor a request for an accommodation; this request should be sent a sufficient amount of time before the officially scheduled due date for the student to learn whether the excuse is acceptable.

Academic Integrity: You are expected to do your own work on all assignments. The minimum penalty for cheating or plagiarism is a failing grade in the course. You will also be reported to the University’s Academic Misconduct Committee. Students are responsible for knowing and understanding the University’s policy on academic misconduct. For further information about this, consult the following website:
https://regulations.utah.edu/academics/6-400.php. Course Policies

Course Schedule

Below, I’ve provided a general outline of the course schedule. However: this syllabus in NOT a binding legal contract. I reserve the right to revise this schedule (including due dates for assignments) as I see fit. If there are any changes, they will be reflected in the official “Course Schedule” on the CANVAS course page.

Weeks 1 - 3: CANTOR, INFINITY, AND SET THEORY

- Historical introduction to Cantor’s life and works
- Introduction to elementary set theory
- Infinity and Cantor’s Transfinite Paradise
- Axiomatic set theory in the twentieth century
- WRITING ASSIGNMENT 1

Weeks 4 - 10: FREGE, RUSSELL, AND LOGICISM

- Historical introduction to Frege’s life and works: The Logicist Project
- Frege’s Begriffsschrift: a revolution in logic
- Frege’s Grundlagen: the definition of Number in purely logical terms
- Function and Concept/Logic: The subject matter of logic
- The Grundgesetze and Russell’s Paradox
- Russell’s Resolution of the Paradox
- WRITING ASSIGNMENTS 2 AND 3

Weeks 11 - 12: HILBERT’S PROGRAM

- Historical introduction to Hilbert’s Program: axiomatic systems and the demand for consistency
- The Frege-Hilbert Controversy
- Hilbert’s Formalism, Model Theory, and Metalogic
- WRITING ASSIGNMENT 4
Weeks 13 - 16: GÖDEL'S INCOMPLETENESS THEOREMS

Proofs of consistency for Principia Mathematica and Lq
Gödel’s proofs and its consequences
WRITING ASSIGNMENTS 5 and 6
Last day of class: Wednesday 12/2

Finals Week: Monday 12/7 – Friday 12/11

NO FINAL EXAM.
WRITING ASSIGNMENT 6 is due before the scheduled due date of the final exam:

Final Exam Time: Friday, December 11, 10:00am.

Other Policies and Resources

Problematic Historical Figure Statement
Historical figures in philosophy, as well as many other intellectual domains, often held horrible views; views that, if held today, would be regarded as abhorrent. The fact is that many notable thinkers from history were racists, sexists, or any number of other terrible things. Some of the figures we will cover this semester are no exception. Fortunately, though, the abhorrent views of these particular figures are completely unrelated to their mathematical/logical/philosophical doctrines — and, in this class, we are exclusively interested in the latter. So, it’ll be easy for us to consider only that which is valuable and leave the rest behind. Still, it’s important to acknowledge both the good and the bad in these figures — and that one’s being an intellectual giant in one domain doesn’t mean that one isn’t a despicable, block-headed fool in another.

Veterans Center
If you are a student veteran, I want you to know that the U of Utah has a Veterans Support Center on campus. They are located in Room 161 in the Olpin Union Building. Hours: M-F 8-5pm. Please visit their website for more information about what support they offer, a list of ongoing events and links to outside resources: http://veteranscenter.utah.edu/. Please also let me know if you need any additional support in this class for any reason.

LGBT Resource Center
If you are a member of the LGBTQ community, I want you to know that my classroom is a safe zone*. Additionally, please know that the U of Utah has an LGBT Resource Center on campus. They are located in Room 409 in the Oplin Union Building. Hours: M-F 8-5pm. You can visit their website to find more information about the support they can offer, a list of events through the center and links to additional resources: http://lgbt.utah.edu/. Please also let me know if there is any additional support you need in this class.

Learners of English as an Additional/Second Language
If you are an English language learner, please be aware of several resources on campus that will support you with your language development and writing. These resources include: the Department of Linguistics ESL Program (http://linguistics.utah.edu/esl-program/); the Writing Center (http://writingcenter.utah.edu/); the Writing Program (http://writing-
program.utah.edu/); the English Language Institute (http://continue.utah.edu/eli/). Please let me know if there is any additional support you would like to discuss for this class.

**ADA**
The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Union Building, 581-5020 (V/TDD). CDS will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability Services.

**Nondiscrimination**
The University of Utah does not discriminate on the basis of race, ethnicity, color, religion, national origin, sex, age, disability, sexual orientation, gender identity, gender expression, genetic information or protected veteran’s status, in employment, treatment, admission, access to educational programs and activities, or other University benefits or services. Inquiries concerning perceived discrimination or requests for disability accommodations may be referred to the University’s Title IX/ADA/Section 504 Coordinator in the Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066.

**Sexual Misconduct/Title XI**
Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran’s status or genetic information. If you or someone you know has been harassed or assaulted, you are encouraged to report it to the Title IX Coordinator in the Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801-581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to the police, contact the Department of Public Safety, 801-585-2677(COPS).

**Wellness Statement**
Personal concerns such as stress, anxiety, relationship difficulties, depression, cross-cultural differences, etc., can interfere with a student’s ability to succeed and thrive at the University of Utah. For helpful resources contact the Center for Student Wellness at www.wellness.utah.edu or 801-581-7776.

**University Safety Statement.**
The University of Utah values the safety of all campus community members. To report suspicious activity or to request a courtesy escort, call campus police at 801-585-COPS (801-585-2677). You will receive important emergency alerts and safety messages regarding campus safety via text message. For more information regarding safety and to view available training resources, including helpful videos, visit safeu.utah.edu.

**Philosophy Resources: (helpful for research and getting a better handle on complex issues)**
- Stanford Encyclopedia of Philosophy: https://plato.stanford.edu/
- Internet Encyclopedia of Philosophy: https://www.iep.utm.edu/